Introduction

Often, you will come across a situation where you need to test whether or not a number is divisible by another particular number. Divisibility tests are short-cuts to determine this, not by actually dividing, but with help of short observations. Following table summarizes and explains these tests.

Divisor	Divisibility Test	Examples
2	Unit Digit is even	12; 24; 42; 66; 88
3	Sum of digits is multiple of three	123 (Since 1+2+3=6 which is multiple of 9); 7629(Since 7+6+9=18 and 1+8=9 which is multiple of 3)
4	Last two digits form a multiple of four	31248 (Since last two numbers form 48 which is multiple of 4)
5	Unit digit is 0 or 5	3195 ; 25; 275; 20
6	Number is divisible by both 2 and 3	36 ; 216 ; 9366120 (Apply tests of both 2 and 3)
7	Difference between sum of alternating blocks of three digits from right is a multiple of 7	1,534,456 (456-534+1)= -77 is divisible by 7
8	Last three digits form a multiple of 8	21248 (248 is multiple of 8)
9	Sum of digits is multiple of 9	1234566 (1+2+3+4+5+6+6=27 which is divisible by 9)
10	Unit digit is 0	10 ; 130 ; 230; 4500
11	Difference of sums of alternate digits is multiple of 11	76542345 => Sum of even placed digits - sum of odd place digits = (7+5+2+4)-(6+4+3+5) = 18-18=0; which is divisible by 11.
12	Number is divisible by both 3 and 4	132 ; 12372 (Apply tests of 3 and 4 both)

Illustration 1

Find all the numbers divisible by 3 between 20 and 40.
Soln. Since, we know that sum of digit of 20 is 2 which is less than 3 by 1, we will get first such number by adding 1. Thus 21 is the first number. Next numbers can be obtained by adding 3 till we reach 40. These numbers are 24, 27, 30,33,36 and 39.

Illustration 2 A number 4532x is divisible by 22. Find value of x?
Soln.

1. To be divisible by 22 the number must be divisible by both 2 and 11. So the number must be even. And it must satisfy divisibility test of 11.
2. Since 4-5+3-2 +x = 0+x = 0 and as x is a number from 0 to 9; we can satisfy divisibility test of 11 only by having x=0
3. Since x=0 , which is even, it is also divisible by 2
4. Hence, number 45320 is divisible by both 2 and 11 (And hence by 22). So value of x is 0.

Illustration 3 If X^2 - Y^2 = 2k where x,y,k are natural numbers, mark following statements as true or false.
Soln.

Statement	True / False
1. Number k is even 2. Either x or y must be even 3. x-y is even 4. x+y is odd 5. x+y-k is even	

Since (x-y) (x+y) is even, either (x-y) or (x+y) is multiple of 2. But, if one of them is even, other must also be even (Think). So, both of them are even.
So, 2k must have 4 as a factor. Or k must have 2 as a factor.

But nothing can be said about x and y except that they are either both odd or both even.

So answers are, TFTFT

Illustration 4 Square of a number is divisible by 12. Cube of the number must be divisible by

1. 8 and 27
2. 8 but not 27
3. 27 but not 8
4. Any of the above may be true.

Soln. Since square of number is divisible by 2^2*3, the number must have both 2 & 3 as factors. Cube of the number will have 2^3*3^3 as factor. Or 216 will be a factor. So, both 8 and 27 will be its factors.

Illustration 5 Square of a number is divisible by 28. Find the highest possible number which must divide the cube of the number perfectly.
Soln.

Since, square of the number is divisible by 28 or 2*2*7; the number must be divisible by 2 and 7.
Cube of the number must be divisible by 2*2*2*7*7*7 = 2744

Is it sufficient to know the divisibility tests and being able to apply them?

You must have understood that divisibility tests are of great help. However, for CAT preparation you should always keep in the mind logic behind the short cuts. Read below to know "Why is it so?"

Once an IIM professor asked a candidate appearing for interview whether he knows divisibility test of 9 and 4. Student explained that sum of the digits of that number must be divisible by 9 for it to be divisible by 9. But before he explained divisibility test for 4, Professor interrupted and asked, "Why such a test is not there for 4? Why I cannot determine the divisibility of 4 by summing the digits of the number and dividing the sum by 4? ".

Do you know the answer to this question?

It is simple; as we studied in an earlier chapter any 3 digit number xyz can be represented as

100x+10y+z
= 99x+9y+(x+y+z).

Since we know that 99 and 9 are divisible by 9, we can certainly say that 99x+9y is divisible by 9 as well. All that we have to check is that whether x+y+z is divisible by 9 or not. And x+y+z is nothing but sum of the digits.

Can you try to explain other divisibility test using similar representations?

Some important results

1. If a number is divisible by another number k, then all factors of k also divide that number completely. For example- 108 is divisible by 12. So 108 will also be divisible by all factors of 12. Hence 108 is divisible by 1, 2,3,4,6 also.
2. If a number is divisible by another number n, then multiples of that number will also be divisible by n. For example, 24 is divisible by 3, so all multiples of 24 i.e. 24, 48, 72 and so on will also be multiple of 3.
3. A number which is multiplication of n consecutive integers is divisible by n. For example (k-1) k (k+1) is multiplication of 3 consecutive integers. So it will be divisible by 3. 15*16*17*18 will be divisible by 4.
4. Sum or difference of two different numbers divisible by n will also be divisible by n. For example- 18 and 24 are divisible by 6, so 18+24=42 will also be divisible by 6.

Practice Problems 1

1. Consider the numbers 2512; 2229; 98172; 7668. Which of the above numbers are divisible by 3? By 4? By 12?
2. For checking divisibility of a number by 36 one should check that the number is divisible by
    1. 4 and 9
    2. 2 and 3
    3. 2 and 9
    4. 4 and 3
3. A number which is divisible by 72 must also be divisible by
    1. 8
    2. 18
    3. 36
    4. All of these
4. A number is divisible by all 11, 2, 3 and 6. The number must be divisible by
    1. 66 (True / False)
    2. 36 (True / False)
    3. 22 (True / False)
    4. 18 (True / False)
5. If n is an integer, which of the Following must be divisible by 3?
    1. (n-1) n (n+1)
    2. (n-2) n (n+2)
    3. (n-3) n (n+3)
    4. Both option 1 and 2
    5. All of the above
6. If sum of digits of a number is divisible by 6 then?
    1. Number is divisible by 2
    2. Number is divisible by 3
    3. Number is divisible by 6
    4. All of the above
7. If the sum of digits is divisible by 111 then the number must be divisible by
    1. 3
    2. 11
    3. 111
    4. All of the above
8. Consider a number abcde48. It is given that the number is divisible by 24. Which of the following must be true?
    1. e=8
    2. a+b+c+d+e is divisible by 3
    3. a+b+c+d+e is divisible by 12
    4. Both 1 and 2
    5. All 1,2 and 3
9. A number divisible by 5 was found to be divisible by 11 and 14 also. Which of the following are true?
    1. Number must be divisible by 7
    2. Number must be divisible by 22
    3. Last digit of the number is 0
    4. All of the Above
10. Sum of the digits of a number is divisible by 15. Then the number must be divisible by
    1. 3 and 5
    2. 15
    3. 3

    4. 5
11. An even three digit number abc is divisible by 11. Which of the following can be the value of a-b+c?
    1. 0
    2. 11
    3. 22
    4. Both 1 and 2
    5. All 1,2,3
12. Two numbers are found to be divisible by 18 and 12 respectively. Which of the following is true about divisibility of the product of these two numbers by numbers 27, 54 and 216
    1. It will be divisible by 27 only.
    2. It will be divisible by 27 and 54, But not by 216.
    3. It will be divisible by all 27, 54 and 216.
    4. None of the conclusions above hold with certainty.
13. Marks obtained by Aman in a test of 100 marks with 25 multiple choice questions carrying equal weight-age are twice that of Raman and Thrice that of Chaman. Chamans marks are twice that of Susheel. Susheel's Marks are Then highest number that must divide Aman's score perfectly is
    1. 6
    2. 12
    3. 24
    4. 48
    5. 96
14. Consider natural numbers A,B,C,D,E. A is divisible by B, B is divisible by C, C is Divisible by D, D is divisible by E. Mark True/False for the following questions

Statement	True / False / Can't say
A is divisible by DE	
AB is divisible by BE	
ABD is divisible by C (E^2)	
A/B is divisible by C/E	
AB > CE	
AD > BE	
D > A	
A > C	

Answers

1.	4. TFTF	7. 1	10. 3	13. 3
2. 1	5. 4	8. 4	11. 4	14. CTTCCCCC
3. 4	6. 2	9. 4	12. 3	

Hints and Explanations

1. Apply divisibility tests of 2, 3, 4 to given numbers.
2. 36 can be written as product of co-prime factors as 2^2*3^3 which is 4*9. Hence, number must be divisible by both 4 and 9.
3. All the options are factors of 72. So, the number will be divisible by all the options.
4. Students should notice that as 6 is itself product of co-prime factors 2&3; it is not necessary for the number to be divisible by product of 2*3*6. Simply put, as both 2 and 6 involve 2^1, only 2^1 will be a factor (And not 2^2)
5. For these question consider following cases
    1. Case 1: n is multiple of 3. In this case all options 1,2,3 will be divisible by 3 since n is a factor in all options.
    2. Case 2: n can be written in form kx+1. In this case reminder obtained by dividing (n-3), (n-2),(n-1),n,(n+1),(n+2),(n+3) by 3 will be 1,2,0,1,2,0,1 respectively. So, for option 1, option 2, option 3 reminders will be 0*1*0 =0, 2*1*0=0, 1*1*1=1.
    3. Case 3: n can be written in form kx-1 (i.e. Reminder 2). In this case reminder obtained by dividing (n-3),(n-2),(n-1),n,(n+1),(n+2),(n+3) by 3 will be 2,0,1,2,0,1,2 respectively. So, for option 1, option 2, option 3 reminders will be 1*2*0=0, 0*2*1=0, 2*2*2=8 (reminder 2), respectively.
    4. Summarily Option 1 and Option 2 always give reminder 0, when divided by 3. But, Option 3 gives reminder 0,1 or 2 in case 1, case 2, case 3 respectively.
    5. So, options 1 and 2 are always divisible by 3. Answer is option 4
6. Both 2 and 3 are co-factors of 6. So, a number divisible by 6 will also be divisible by both 2 and 3.
7. 3 is a factor of 111. So, If sum of the digits is divisible by 111, then it will also be divisible by 3. Thus, divisibility test of 3 is satisfied. Fot other options there is no test based on sum of digits.
8. Let us consider each option individually
    1. By applying Divisibility test of 8; e24 should be divisible by 8, here e may be both 0 and 9. So, option 1 is not necessarily true.
    2. abcde48 can be written as abcde(100)+ 48 = abcde + abcde(99) + 48. Now since abcde(99) + 48 is divisible by 3, For abcde48 to be divisible by 3, abcde must also be divisible by 3. So answer is Option 2.
    3. abcde(100) + 48 is divisible by 24; so abcde(100) must be divisible by 24. Since, 100 has 4 as a factor, abcde must be divisible by 6 (and not 12). For example consider number 1233048 which is divisible by 24. But 12330 is not divisible by 12.
9. 5, 2,7,11 must be factors of such number. Hence, it will be divisible by all 7, 22, and 10. Hence, answer is all of the above.
10. Sum of the digits tell us about divisibility by 3 does not tell anything about divisibility by 5. Hence, Option 3.
11. Since all a,b,c have 9 as maximum possible value. a-b+c cannot be 22. Option 1 and Option 2 are correct.
12. 18=2*3^2 and 12 = 2^2*3. Since, we are multiplying the number; power of co-factors will be added. Hence, number will be divisible by 2^3*3^2 = 216. Option 3.
13. Each question carries 4 marks. Let A, R, C, S be marks of the students respectively. We can relate them as follows.
    1. A=3C
    2. A=2R
    3. C=3S
    4. We get A= 6S from 1 and 3
    5. But S itself is multiple of 4, say 4x.
    6. So A= 6S = 24x. A must be multiple of 24.
14.

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AB is divisible by BE	Can't say.
ABD is divisible by C (E^2)	True.
A/B is divisible by C/E	Can't say.
AB > CE	Can't say. Consider perfect equality A=B=C=D=E ;it g = CE
AD > BE	Can't say. Consider perfect equality A=B=C=D gives AD=BE
D > A	Can't say. Consider perfect equality A=B=C=D gives D=A
A > C	Can't say. Consider perfect equality A=B=C=D=E ;it gi C

Explanations

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AD > BE	Can't say. Consider perfect equality.
D > A	Can't say. Consider perfect equality.
A > C	Can't say. Consider perfect equality.